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Free convection in low-temperature gaseous helium

By D. C. THRELFALL
Cavendish Laboratory, University of Cambridge

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Free convection has been studied in gaseous helium at low temperatures in a cylindrical vessel of aspect ratio (diameter/height) 2·5. Compared with measurements in fluids at room temperature the present arrangement has the advantages of small size, a short time constant and improved accuracy. As the Rayleigh number was varied from 60 to 2×10^9, the Nusselt number rose from 1 to 69, obeying the relation $Nu = 0{\cdot}173 Ra^{0{\cdot}2800 \pm 0{\cdot}0005}$ over the upper four decades of Ra. The critical Rayleigh number was 1630, but the conditions of the experiment did not allow reliable measurements at such low values of Ra. The very high sensitivity within a given experiment showed the presence of several 'discrete transitions', which were often step like and not merely a change of gradient as reported by other workers. The largest of these, at $Ra = 3 \times 10^5$, involved a drop in heat flux of some 6 % and was somewhat hysteretic. The temperature fluctuations increased markedly as the step was crossed.

1. Introduction

When heat is convected vertically upwards between infinite horizontal plates a distance L apart owing to a temperature difference ΔT, the conditions can be characterized by the dimensionless Rayleigh number

$$Ra = L^3 \rho^2 g C_p \beta \Delta T / k\mu, \tag{1}$$

where ρ is the density of the fluid, g is the acceleration due to gravity, C_p is the specific heat per unit mass, β is the expansion coefficient, k is the thermal conductivity and μ is the dynamic viscosity. The heat flow can be described in dimensionless terms by the Nusselt number, which is the ratio of the actual heat flow to that which would occur if only conduction took place and convection were absent. In the simplest form of the theory of convection Nu is a universal function of Ra, though if the plates are of finite diameter D the aspect ratio D/L also plays a part in determining the behaviour, albeit a small part if D/L is large:

$$Nu = F(Ra, D/L).$$

This paper is primarily concerned with presenting an experimental technique which, it is believed, gives considerably higher accuracy than earlier work as well as making accessible a wider range of Ra. Only one aspect ratio has been investigated fully, but this is enough to reveal the complexity of the behaviour and the need for equally systematic studies of other aspect ratios. Although, therefore, the results are presented as if Nu were a function of Ra alone, it will be appreciated that D/L is a hidden parameter, and that it is premature to attempt detailed comparison with other work in which a variety of aspect ratios were employed.

The larger the Rayleigh number, the more effective is the convective process, and the present experiments, which made use of helium gas at temperatures around 4 °K, were designed to achieve very large values of Ra. k and μ in helium are very low, the density is much higher than for most gases at N.T.P. while β, being $1/T$ for a perfect gas, is abnormally large. So $Ra = 10^9$ is possible when L is only a few cm. A similar Ra at room temperature would require plates up to a hundred times further apart. At the same time, by reducing the gas pressure the Rayleigh number can be made as small as desired, so that the whole range from conduction to extreme convection can be investigated in a single experimental cell.

One benefit of conducting the experiment at low temperatures is that the convective cell may be suspended in a high vacuum, eliminating heat conduction to the experiment. Heat input due to radiation may be removed by screening.

The virtually perfect isolation of the sides of the cell opens the way to the study of small aspect ratios, and preliminary results show that there is indeed a significant change as D/L is reduced. A further bonus of low temperatures was the low thermal capacity of the materials of the cell, which allowed rapid response of the thermometer to fluctuations in the gas temperature and thereby provided information not normally available in this type of experiment.

The best measurements to date in the turbulent region are those due to Goldstein & Chu (1969) and Chu & Goldstein (1973), which were done with air and water as the working fluids. In these experiments the temperature gradient was inferred from Mach–Zehnder interferometer measurements. For water, a scatter of less than 5 % was achieved with a probable error of $< \frac{1}{2}$ % in the gradient of a logarithmic plot of Nu against Ra, which covered three decades of Ra. At certain values of Ra they found evidence of kinks of the kind that are usually interpreted as transitions in the mechanism of convection. These were first studied by Malkus (1954) and have been the subject of still more refined investigations by Goldstein and Chu as well as Brown (1973), Krishnamurti (1970) and Willis & Deardorff (1967), but there is considerable disagreement in detail.

Some of the disagreement may be due to different workers using different aspect ratios, but just as important is the great difficulty of discerning discontinuities in the presence of experimental scatter. The present results show how much more easily accurate and reproducible results may be obtained with helium at low temperatures. A systematic study using this approach might go a long way towards developing a definitive picture.

2. Apparatus

The apparatus followed standard low-temperature practice. Figure 1 shows the arrangement in the working space of the cryostat. The experimental can was made from low-thermal-conductivity copper–nickel alloy of internal diameter 48·4 mm and thickness 0·2 mm. The parallel end plates, which were recessed into the can to eliminate any round edges from soldering, were of oxygen-free high-

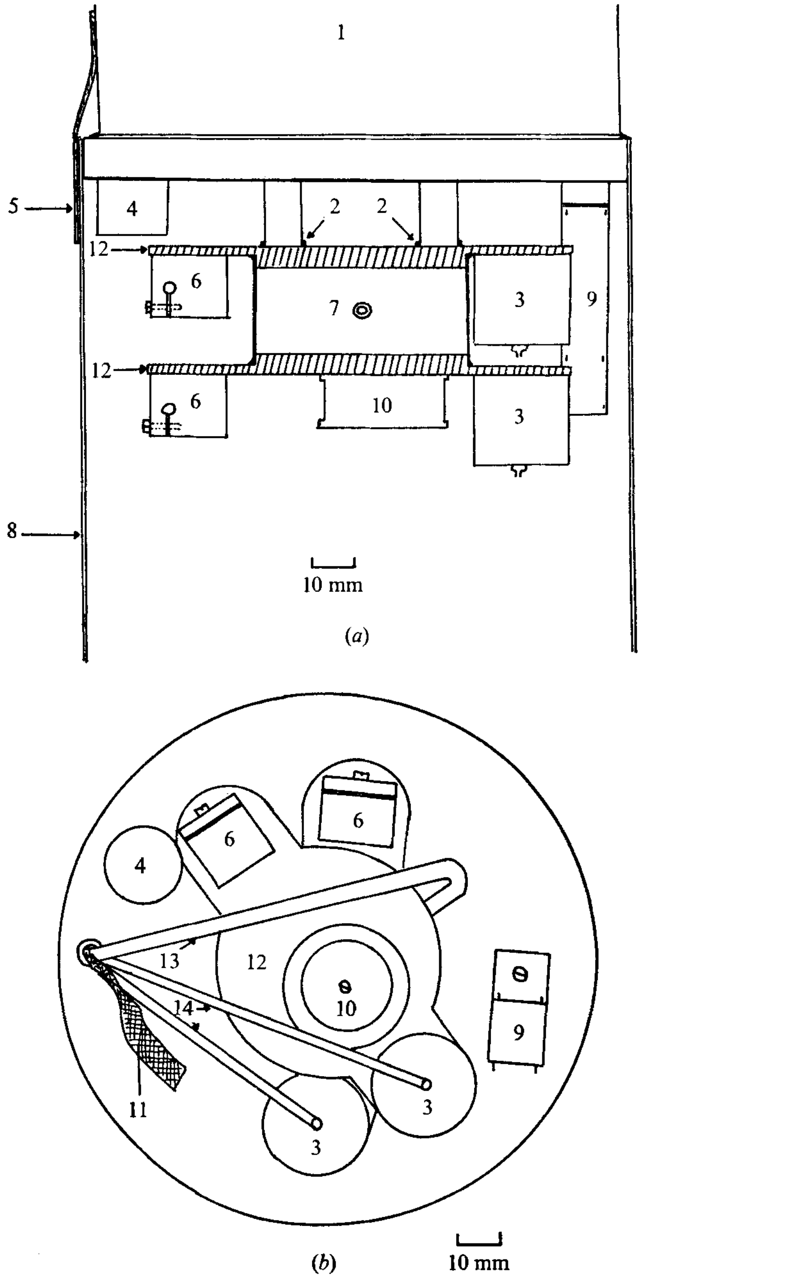

FIGURE 1. (*a*) Elevation of the experimental space of the cryostat. (*b*) View of the experimental space from below. 1, liquid-helium space; 2, indium joints to the two copper rods let into the helium space; 3, gas-thermometer bulbs; 4, bollard to heat sink the wires; 5, braids (two) soldered to the screening can and the liquid-helium can; 6, copper blocks containing germanium thermometers; 7, experimental helium space; 8, screening can; 9, heat sink/tag board for wires; 10, copper former for heater wound with Eureka wire; 11, braid thermally anchoring tubes; 12, copper end plates recessed into the copper–nickel can; 13, 3 mm bore tube to the experimental space (omitted from (*a*) for clarity); 14, 1 mm bore tube to gas thermometers (also omitted in (*a*)).

conductivity copper (measured thermal conductivity > 1 W/cm °K). The top plate was 5 mm thick and the lower plate was 6 mm thick. They were 19·96 mm apart inside the can and were milled so that lugs for thermometry would be integral. The method of anchoring the can to the helium bath ensured that the upper plate was only a little warmer than the bath.

The absolute temperature of the top plate was measured with one germanium thermometer, and the pair of germanium thermometers, one at each end, acted as a cross-check on the gas thermometer used for measuring ΔT. On the bottom of the can was a small heater of copper wound with Eureka resistance wire. This heater was mounted slightly off centre to allow for a thermometer inside the can, a facility not used in this experiment. This eccentricity led to a temperature gradient across the plate of less than $2\frac{1}{2}$ % of ΔT.

Extraneous heat flow into the experiment was kept to a minimum by thermally anchoring tubes and wires, and by some use of superconducting wires. The unwanted heat input was estimated from measurements as $10\,\mu$W. This is negligible when $Ra > 10^4$ and produces a 5 m °K temperature rise when $Ra = 10^3$.

The tube admitting gas to the experimental can, which was also used for measuring the gas pressure, was 3 mm in diameter, wide enough for thermomolecular pressure differences to be very small, even at 0·1 cm Hg, the lowest pressure used in this experiment. According to the form of thermomolecular pressure drop used by Hulm (1950), derived from Weber, Keesom & Schmidt (1936), the maximum error is less than $\frac{1}{2}$ %. The gas entered the can through a hole of diameter 1 mm which is believed to have no significant effect on the flow inside. The tubes for the gas thermometer were 1 mm in diameter. This represents a compromise between thermomolecular pressure errors and dead space.

3. Experimental measurements

Heat

The heat supplied to the experiment was measured with a four-terminal system. The current and voltage were read with similar digital multimeters accurate to within about 0·2 % full scale.

Absolute temperature

A CRYOCAL 1000 germanium resistance thermometer was calibrated by Cryogenic Calibrations against a T_{58} scale substandard (Brickwedde *et al.* 1960). Including a spline fit the calibration was found to be within 10 m °K when compared with the helium bath temperature. The smoothness of the interpolated curve should be of the order of 1 m °K. With a $3\frac{1}{2}$ digit digital voltmeter to read the voltage developed across the thermometer owing to the $10\,\mu$A measuring current, the resolution of $10\,\mu$V represented a random error of ± 2 m °K.

Mention should be made of the temperature scale used here. The T_{58} scale, which follows standard low-temperature practice in using the vapour pressure of liquid helium as the thermometric property, is now known to be in error by about 10 m°K at 4·2 °K and a new scale has been proposed by McCarty (1972). As no new standard has been accepted, all temperatures referred to here are on the T_{58} scale. The difference is smooth and will not affect the shape of the curves shown.

Temperature difference

A differential gas thermometer was used with oil to indicate the pressure difference. It consisted of two small vessels at 4 °K and a miniature oil manometer at room temperature connected by fine tubes. The method was originated by Hulm (1950). The temperature difference ΔT between the two small vessels and the resulting pressure difference ΔP are related by the approximation

$$\Delta P/P = \Delta T/T,$$

where P and T are the absolute pressure and temperature in the vessels at the time of filling (after which time they are isolated from each other). A total correction of about 5 % was necessary to allow for the dead spaces (volumes not involved in the temperature change), non-perfect gas relations and thermomolecular pressure differences (usually negligible at the pressures and temperatures involved here). The system was very insensitive to changes in the mean temperature, a deflexion of less than 1·5 m °K being seen after a 0·81 °K change in ambient temperature. For a fuller description the reader is referred to Hulm's (1950) paper and thesis (1949).

The pressure difference was read to within 10 μm of oil with a vertical travelling microscope, which could be swung between the two arms of the manometer. The absolute pressure used was chosen to suit the sensitivity and maximum ΔT required. Usually a sensitivity of 100–200 μ °K was used with ΔT in the range 20–500 m °K. The ratio between the maximum and minimum ΔT was about 30 within each run.

The absolute pressure of the gas was measured using the same oil as in the manometer so that the density of the oil cancelled to first order. The absolute pressure was normally 20–40 cm of oil and could be measured to an accuracy of $> \frac{1}{2}$ % with ease. The Boussinesq equations demand that the adiabatic lapse across the can be taken into consideration; however, this is about 4×10^{-5} °K in this case and is neglected.

Can pressure

Pressures greater than 1 cm Hg were measured on a mercury manometer and those less than 1 cm Hg on a Vacustat gauge. The accuracy of the latter was no better than 2 % although the overlap in measurements with those taken on the mercury manometer appeared to be satisfactory. As the low pressures implied low Ra and the experiment was designed for high Ra no attempt was made to improve the low-pressure measurements, especially in view of errors resulting from the slight heat leak.

Thermophysical properties

These were all taken from the best set available (McCarty 1972). A version of this was supplied as a computer program by Hands (1972, 1973). McCarty tabulates μ, k and C_p from smoothed experimental data. ρ comes from the P, V, T surface mentioned in the same report and β was obtained from the derivatives:

$$\beta = \frac{1}{\rho}\left\{\frac{\partial P}{\partial T}\right\}_\rho \Big/ \left\{\frac{\partial P}{\partial \rho}\right\}_T.$$

Systematic errors are still possible and in some cases may be as much as 5 %. k and C_p may also suffer from errors in the temperature scales used, as they rely on the gradient of the scale. Owing to the difference between T_{58} and McCarty's scale the McCarty equivalent of T_{58} was calculated and the required property was then found. Typical values are $k = 20T\,\mu$W/cm °K at 0·1 atm, $\beta = 0{\cdot}233$ °K^{-1} at 4·5 °K and 0·1 atm (β varies as $1/T$), and $\mu = 11{\cdot}5\,\mu$P at 0·1 atm and 4·5 °K (μ varies as $\sim T^{0\cdot 9}$ with a small pressure dependence).

Can correction

The heat conducted by the can was measured over the relevant temperature range and for example was 0·67 mW/°K at 4·2 °K; this should be compared with the 0·90 mW/°K which would be transferred by conduction in the gas. The relatively high flux through the walls may upset low-Nu measurements.

Experimental method

Two methods of measurement were employed. In the first, heat was applied to the lower plate and the bath temperature reduced by pumping until the lower germanium thermometer indicated that the lower plate had regained its initial temperature. In the second, the bath temperature was reduced only enough to keep the top plate as its initial temperature. In both cases the temperature difference and the mean temperature (at which all the properties were calculated) were deduced from the gas-thermometer readings. The function of the germanium thermometers was to indicate that the temperatures were constant. Usually the second method was more convenient but as far as the results were concerned there was no significant difference.

The increase of heat in each case was step like and ΔT etc. were not read until the temperatures were constant. The time required for ΔT to settle was 5–20 s, and 3–5 min were allowed between readings. In all the experiments, except that on hysteresis, ΔT was increased. All the readings taken in each run have been plotted except in the main part of figure 2, where about half were omitted for clarity.

4. Results

Many runs were performed using this system, and the results are shown in figure 2, each symbol representing a different run and pressure. The range of Ra can be seen to be very large, extending from 60 to 2×10^9. The large range was obtained by varying the pressure P of the gas and the temperature difference ΔT (Ra varies roughly at $P^2\Delta T$). The groups overlap to test consistency and independence of the variation of properties. Even with the plates at 4·2 and 5·4 °K ($\Delta T = 1{\cdot}2$ °K) and $P = 20$ cm Hg, no appreciable deviation could be seen even though the P, T relation is nonlinear and (1), defining Ra, should begin to be inadequate. However no ΔT greater than 800 m°K was used in the results shown here. Since the same cell was used throughout, D/L was constant at a value of 2·5.

At high values of Ra, a general form $Nu = kRa^m$ has been assumed for many years and for values of Ra above 4×10^5 this equation applies well with

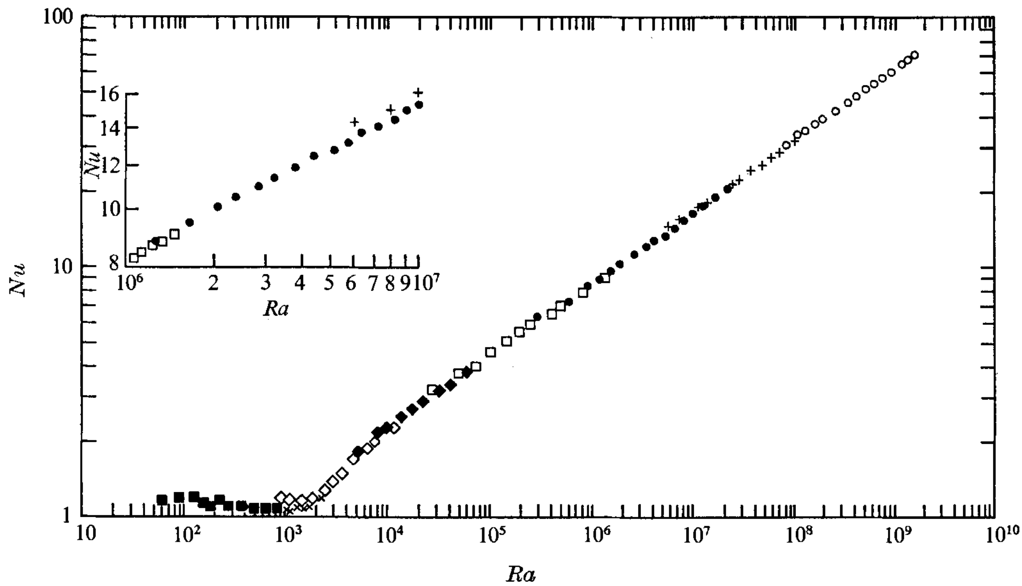

FIGURE 2. Summary of the heat-transfer experiments; $60 < Ra < 2\times10^9$. Above $Ra = 4\times10^5$, $Nu = 0{\cdot}173Ra^{0{\cdot}2800}$; the inset shows the worst scatter in this region with all points plotted.

$k = 0{\cdot}173 \pm 0{\cdot}002$ and $m = 0{\cdot}2800 \pm 0{\cdot}0005$; only 4 of the 95 points deviate from the line by more than 2%. The value of the exponent m is significantly different from the value $\frac{1}{3}$ obtained in early experiments (e.g. Globe & Dropkin 1959) but is very close to the value 0·278 found recently by Chu & Goldstein (1973), who used water with D/L ranging from 1·5 to 6, similar to ours. Indeed the present results lie within $2\frac{1}{2}$ % of Chu & Goldstein's when $Ra = 10^8$ and appear to justify the neglect (at large Ra) of the possible role of the Prandtl number $C_p\mu/k$, which is 6·8 in water and about 0·8 in helium. (At 0·1 atm, $Pr = 0{\cdot}66$ at 3·5 °K and 0·65 at 4·5 °K; at 0·9 atm, $Pr = 0{\cdot}91$ at 4·5 °K and 0·82 at 5 °K.) But note that preliminary results indicate that, at $Ra = 10^8$, Nu lies 25 % above the curve of figure 2 for $D/L = 0{\cdot}33$ and 75 % above when D/L is 0·14.

The linear exponent is only appropriate above $Ra = 4\times10^5$ since for lower values the curve has various irregularities (a very small irregularity at 5×10^6 has been glossed over but is, nevertheless, discernible). At $Ra = 3{\cdot}2\times10^5$ a large discontinuity appears (figure 3), the heat flux drops significantly and at the same time the fluctuations in ΔT increase suddenly. Transitions have been observed by Malkus, Chu & Goldstein, Krishnamurti and Brown but are much more readily apparent in the present observations. One of the germanium thermometers was used to look at the oscillations of the temperature of the lower plate (no oscillations being seen on the upper plate, which was in close thermal contact with the bath) and the maximum deviation from the mean was about 6 m °K. These oscillations showed up clearly because of the low thermal capacity of the lower plate. Figure 4 shows several traces of temperature against time as the transition was passed. Figure 5 shows the r.m.s. amplitude of the signal as a

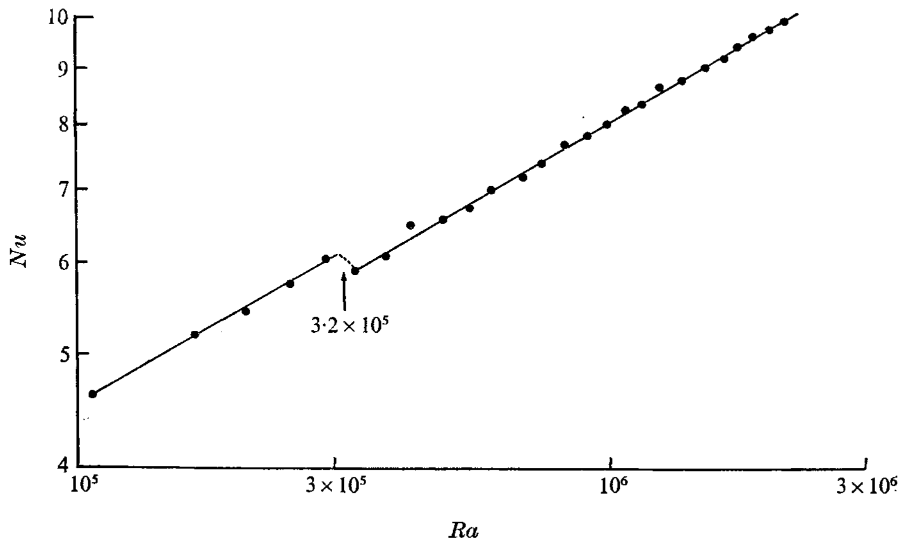

FIGURE 3. The point of the largest discontinuity in heat flux, at $Ra = 3{\cdot}2 \times 10^5$.

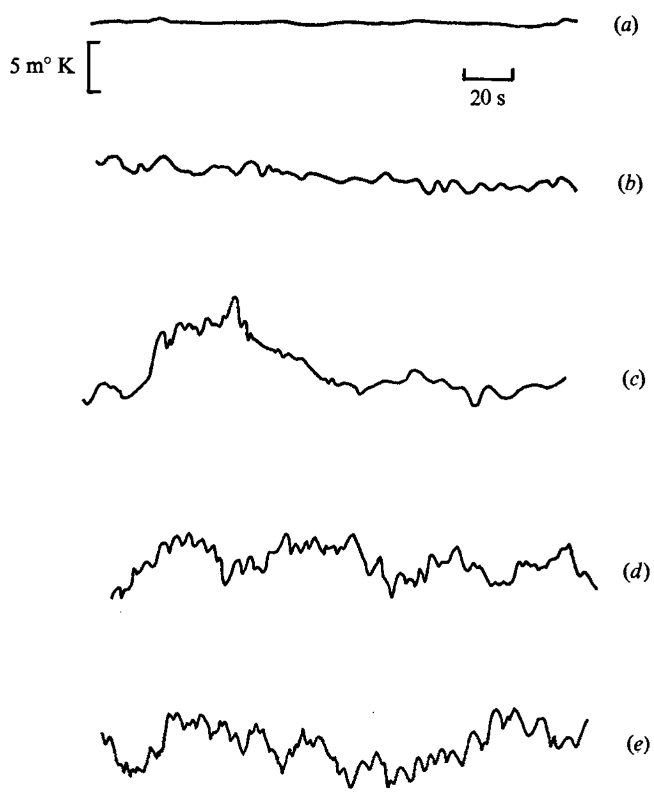

FIGURE 4. The signals seen on the lower plate near the transition in figure 3. (a) $\Delta T = 0$. (b) $\Delta T = 143$ m°K, $Ra = 2{\cdot}7 \times 10^5$. (c) $\Delta T = 165$ m°K, $Ra = 3{\cdot}2 \times 10^5$. (d) $\Delta T = 178$ m°K, $Ra = 3{\cdot}5 \times 10^5$. (e) $\Delta T = 199$ m°K, $Ra = 3{\cdot}8 \times 10^5$.

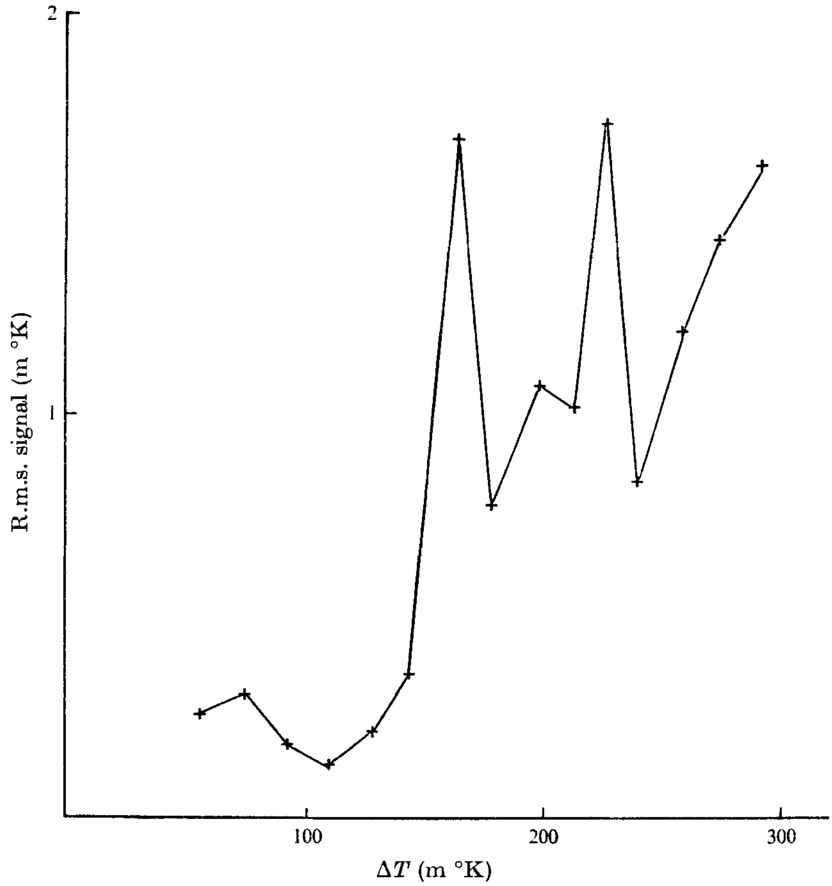

FIGURE 5. R.m.s. amplitude of the signal on the lower plate near the transition in figure 3.

function of ΔT ($\propto Ra$). Changing the pressure by 50 % in repeat experiments was found to have no appreciable effect on the Rayleigh number of the transition. The change in heat flux was hysteretic (figure 6), not occurring at the same Ra when ΔT was increased as when it was decreased. The magnitude of the drop was about 6 %. At A the system appeared to be almost unstable.

Other changes of gradient and discontinuities were seen. The three in figure 7 were confirmed by more than one experiment and others were seen at different values of Ra but not reproduced. The form of the ordinate variable is relevant; by eliminating the general gradient of the plot of $\log Nu$ *vs.* $\log Ra$, in this case a 0·25 power law, the transitions become much clearer, and the reliability of each point is more apparent than in other presentations which plot $RaNu$ against Ra.

The two lower transitions at $Ra = 9000$ and 28000 agree well with those seen by Brown (1973) with an aspect ratio $D/L > 12$. Figure 8, in which the lower range of Ra was studied with especial care, gives support to the view that both are discontinuous; this is particularly clear at the lower transition ($Ra = 9000$).

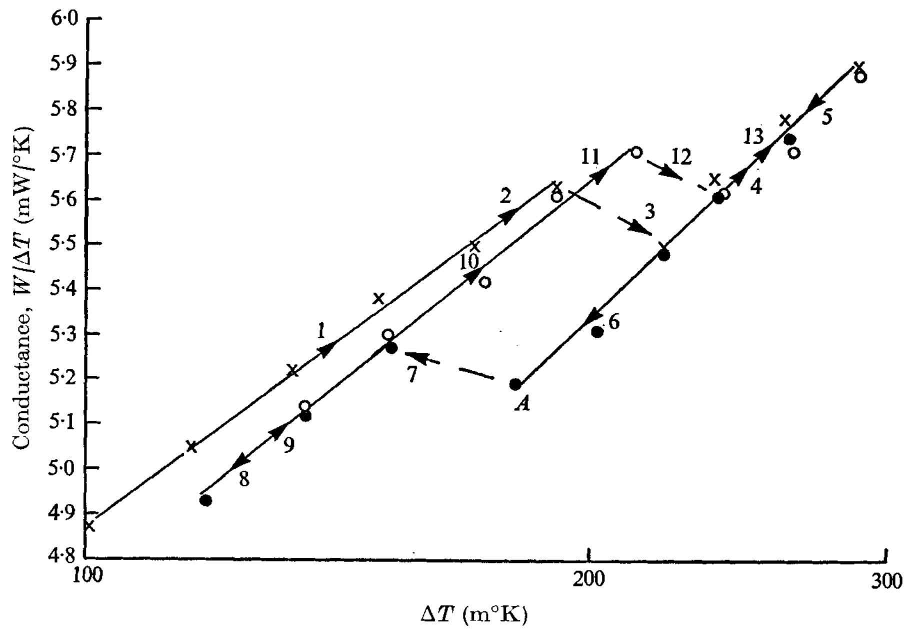

FIGURE 6. Conductance as a function of log ΔT, showing hysteresis around $Ra = 3{\cdot}2 \times 10^5$. All the points are from one experiment; the numbers indicate the order. $\times$, ΔT increasing; ●, ΔT decreasing; ○, ΔT increasing again.

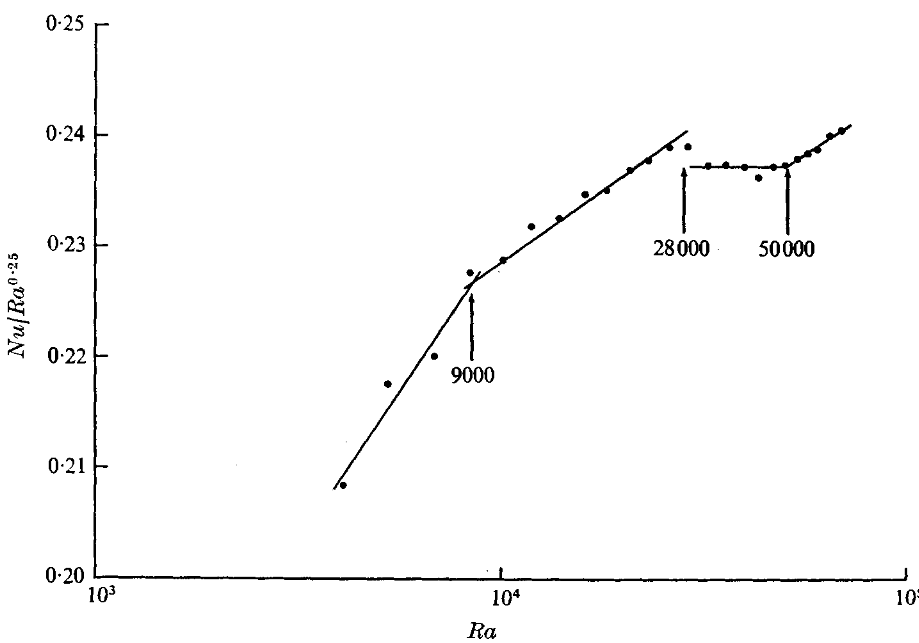

FIGURE 7. Three more transitions at lower Ra.

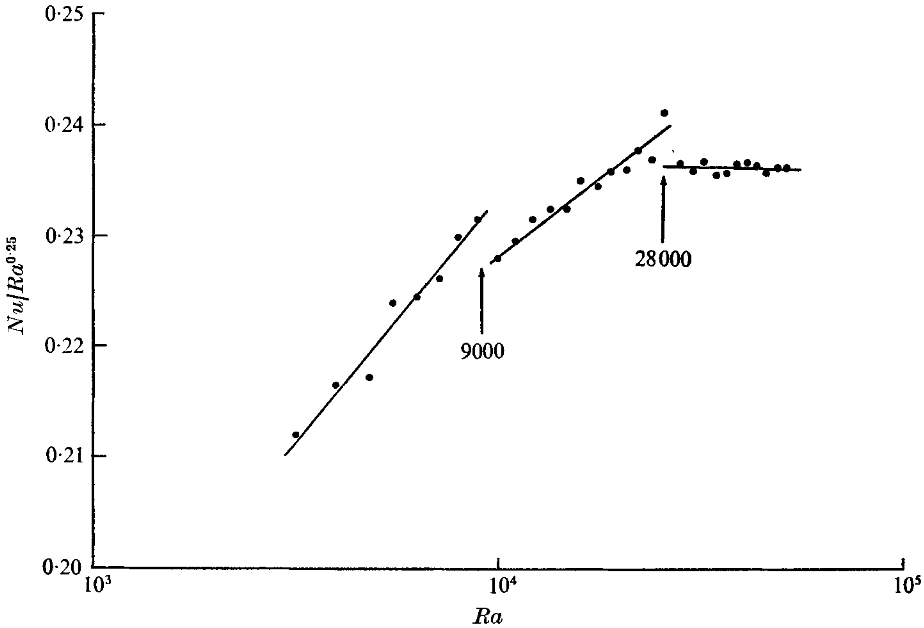

FIGURE 8. Discontinuous nature of the transitions at $Ra = 9000$ and 28000; in this case the sensitivity around $Ra = 9000$ was higher than at the same Ra in figure 7.

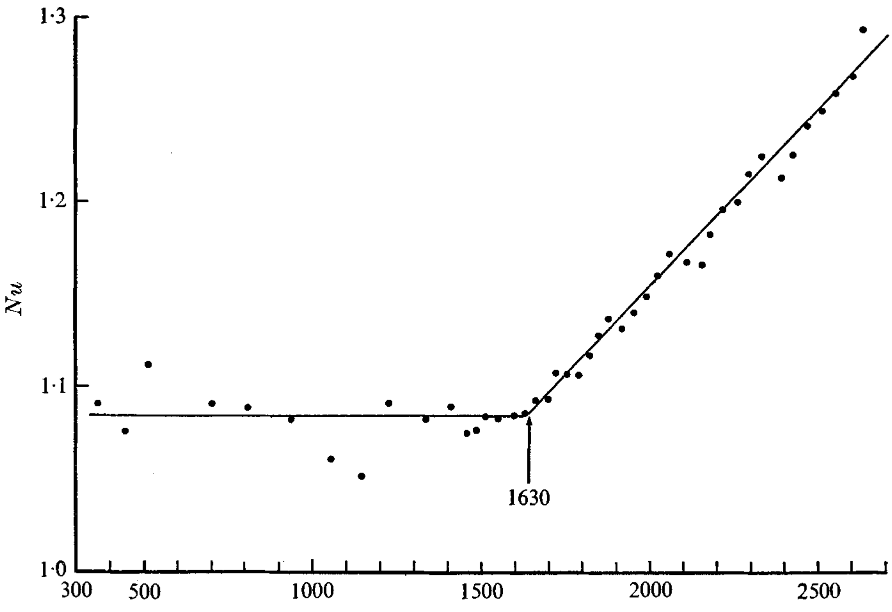

FIGURE 9. Variation of Nu with Ra near the onset of convection; $Ra_c = 1630$.

No discontinuity was claimed by Brown, although one set of points on his figure 5 can be reasonably interpreted as showing this, as well as a further kink at $Ra = 50\,000$. In the present work the transitions confirmed by more than one experiment were at 9000, 28000, 50000, 65000 and $3{\cdot}2 \times 10^5$. Slight changes of gradient were seen, but not confirmed, at $1{\cdot}1, 1{\cdot}6, 2{\cdot}3 \times 10^5$ and 5×10^6. These may represent further transitions.

The other major gradient change, not mentioned so far, was the transition from conduction to convection. This was investigated on several occasions; figure 9 shows one such experiment. The transition can be seen to be sharp and to define Ra_c as about 1630. In the conduction region the calculated value of Nu was 1·08 not 1·00 as of course would be expected. A possible source of error is the thermal conductivity but unfortunately McCarty is unclear in stating his sources for data around 4 °K. The conductivity of the can would need to be more than 10 % wrong to give the required correction, which again seems unlikely. However, as the experiment was designed for the high-Ra regime this was not pursued any further.

It is a pleasure to take this opportunity to thank Professor A. B. Pippard for many hours of helpful discussion, the technical staff for their help in the construction of the experiment, Mr B. A. Hands for providing 'HEPROP', containing the thermophysical properties, and Dr D. O. Gough and Dr N. O. Weiss for critical appreciation and discussion. I am grateful to the CEGB for sponsoring the project and to the SRC for a studentship.

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